“Forecasting the geomagnetic secular variation by machine learning”

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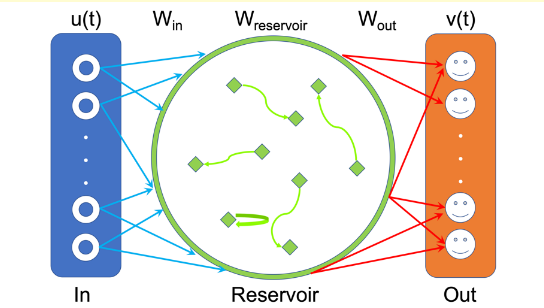
Echo State Network (ESN) Method

Echo State Network is a variant of reservoir computing applicable to machine learning for time-series prediction.



The so-called geomagnetic secular variation (GSV) is, at a glance, dominated by smooth and gradual changes in the vector geomagnetic field. However, it still contains highly non-linear temporal variations such as the geomagnetic jerk characterized by abrupt changes in the first time derivatives of, especially, the eastward geomagnetic components. Presence of those non-linear variations makes the prediction of our planet's magnetic field difficult enough to force the geomagnetic research communities revise their global geomagnetic field models unexpectedly urgently (e.g., Witze, 2019).

Conventional prediction of GSV has been made empirically taking advantage of the linearity of its principal element. However, this doesn't work especially when the aforementioned geomagnetic jerks occur. To circumvent this, the number of GSV predictions using a combination of geodynamo simulation and data assimilation is increasing recently (e.g., Minami et al., 2020). The combination turned out effective for non-linear predictions of GSV except for its computer-intensive nature in both geodynamo simulation and data assimilation. Namely, it is not until working with an enough number of ensemble members (each of which is a result of a fairly long geodynamo run) that data assimilation for GSV forecast becomes effective.

On the other hand, machine learning is an emerging technique, which has a promising potential of application to highly non-linear processes such as the geomagnetic jerks. For example, Koopman Mode Decomposition is a variant of machine learning methods based on a decomposition of observed time-series into multiple modes, while the key procedure of Recurrent Neural Network is feedback of various internal states to the present status. In this study, we adopted ESN (e.g., Nakano and Kataoka, 2022) to perform hindcasts of short-term GSVs from 2005 and 2010 using a geomagnetic field model (COV-OBS; Gillet et al., 2013) stemming from 1840 because ESN is known as cost-effective reservoir computing and works even with relatively few training data.

Conclusions

- We conducted the hindcasts of GSV for Epochs 2005 and 2010 using ESN, one of the simplest reservoir computing methods in machine learning. In doing so, we adopted the COV-OBS (Gillet et al., 2013) geomagnetic field model since 1840 as ‘observation’.
- ESN turned out so effective that it was able to reproduce the short-term GSV without longer training data with a typical 10-y mean prediction error of less than 100 nT.
- It was found that the data covariance may have a large impact in forecasting GSVs.
- It, however, may not be possible to forecast occurrence of geomagnetic jerks even by the machine learning method used here. This needs further elaboration such as inclusion of Length of Day in input data.

Results

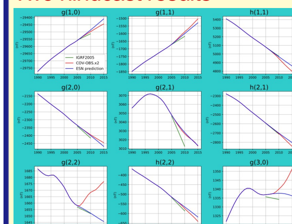
ESN was found so effective for short-term forecast of GSV that it didn't need long training data.

Table 1. Hindcast errors of ESN as a function of training data length and time resolution of the COV-OBS geomagnetic field model.

Length of training data Time resolution [year]	1840 - 2005 0.25	1840 - 2005 0.5	1840 - 2005 1	1900 - 2005 0.25	1900 - 2005 0.5	1900 - 2005 1
Estimation Error [nT] (10-y average)	95	**105**	105	74	**81**	102
Estimation Error [nT] (After 10 yrs)	238	**242**	238	186	**171**	227

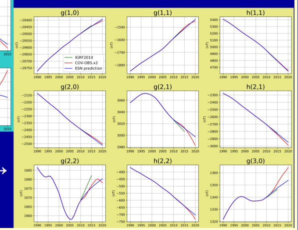
Table 1 clearly shows that ESN doesn't require longer training data for prediction of short-term GSVs of our planet's main magnetic field. Rather, the shorter training data of 105 yrs yielded better accuracy. The dependence of time resolution, i.e., the total number of data, is weak enough to give the best estimates by a combination of shorter training data and half a year resolution. The improvement in estimation error sums up to 20 to 30 %.

Two hindcast results



← Hindcast for 5yrs from Epoch 2005. First 9 Gauss coefficients are shown.
RED = COV-OBS.x2

Hindcast from Epoch 2010.
BLUE = Prediction by ESN →
GREEN = IGRF-SV

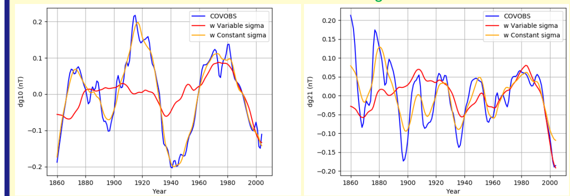


The panel above compares the hindcast results for Epoch 2005 (Left) and 2010 (Right). Although overall forecasts are fairly good for both epochs, ESN seems to have failed to follow abrupt changes in the internal Gauss coefficients (e.g., g22 at around 2005). The hindcast, therefore, does not necessarily guarantee the flexibility of ESN to reproduce non-linear temporal variations such as the ‘geomagnetic jerks’.

Discussion I:

The objective function of this study: $J = \sum_{k=1}^{K} \frac{\left\| \boldsymbol{d}_k - \boldsymbol{\Gamma}^{\mathsf{T}} \boldsymbol{x}_k \right\|_2^2}{\sigma_k^2} + \frac{\|\boldsymbol{\Gamma}\|_F^2}{\lambda^2}$

Data covariance does affect fits of the machine learning method to the observation.

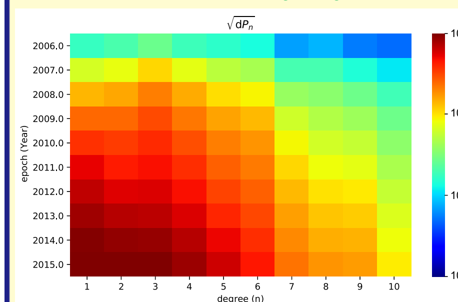


The objective function, ***J***, of this study to determine the matrix $\boldsymbol{\Gamma}$ containing W_{out} is given by $J = \sum_{k=1}^{K} \frac{\left\| \boldsymbol{d}_k - \boldsymbol{\Gamma}^{\mathsf{T}} \boldsymbol{x}_k \right\|_2^2}{\sigma_k^2} + \frac{\|\boldsymbol{\Gamma}\|_F^2}{\lambda^2}$, where σ_k is the so-called ‘data covariance’. One interest is how the data covariance affects the result. The figure above shows it in terms of fits to two specific trained differential Gauss coefficients, $\Delta g10$ and $\Delta g21$, from 1860 through 2005. Although constant σ_k‘s seem to follow COV-OBS (the blue solid line) much better, it does not necessarily mean that the variable σ_k‘s are worse. Because older data are likely to have larger observation errors, it is rather desired to minimize the effect of older data with large errors. ESN seems indifferent to data older thatn 1960, whereas it neatly follows the differential COV-OBS model for the latest years (1995 and onwards).

Discussion II:

The degree-dependent forecast error: $\sqrt{dP_{n,k}} = \sqrt{\sum_{m=0}^{n} (n+1) \left[\left(g_{n,p}^m(t_k) - g_{n,o}^m(t_k) \right)^2 + \left(h_{n,p}^m(t_k) - h_{n,o}^m(t_k) \right)^2 \right]}$

Smaller forecast errors for higher-degree Gauss coefficients



It's quite natural that the forecast errors increase with time. However, they do possess degree-dependence because the higher degree Gauss coefficients vary more rapidly, which does appear even in short teaching data.

Acknowledgements

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