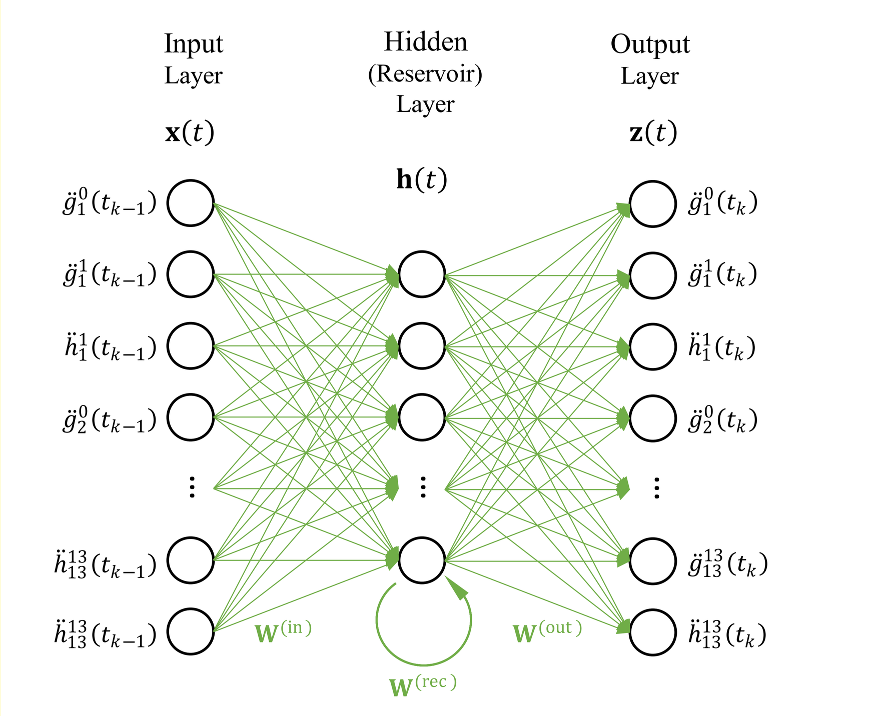
In this study, we utilized an RNN to forecast geomagnetic SV (**Table 2**) for the period from 2025 to 2030, employing the EKF algorithm for training.

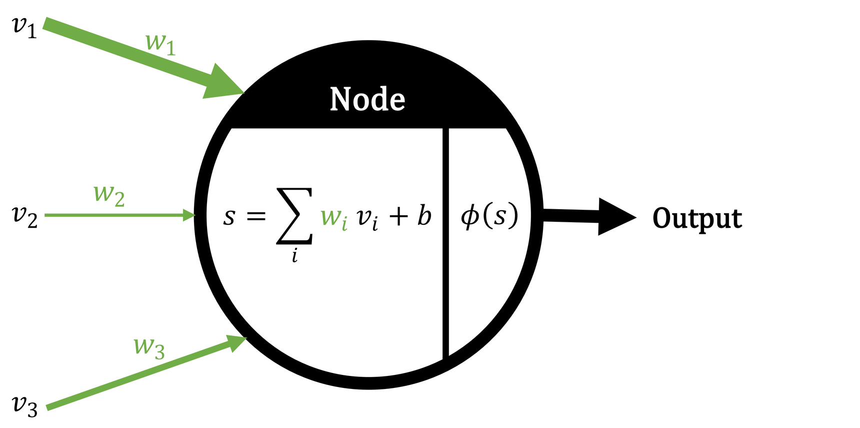
"Forecasting the geomagnetic secular variation by machine learning"

H. Toh, S. Sato [Division of Earth & Planetary Sciences, Kyoto University] &
S. Nakano [The Institute of Statistical Mathematics; Center for Data Assimilation Research and Applications, ROIS-DS]

New Recurrent Neural Network Method with an Extended Kalman Filter as Training Interface

Recurrent Neural Network realizes time-series prediction by recursively updating weights ($W^{(*)}$'s) among neurons.





Hidden Layer (Reservoir)

$$\mathbf{h}_k = \tanh\left(\mathbf{W}^{(\mathrm{in})} \cdot \mathbf{x}_k + \mathbf{W}^{(\mathrm{rec})} \cdot \mathbf{h}_{k-1} + \mathbf{b}^{(\mathrm{rec})}\right)$$

Output Layer (Affine)

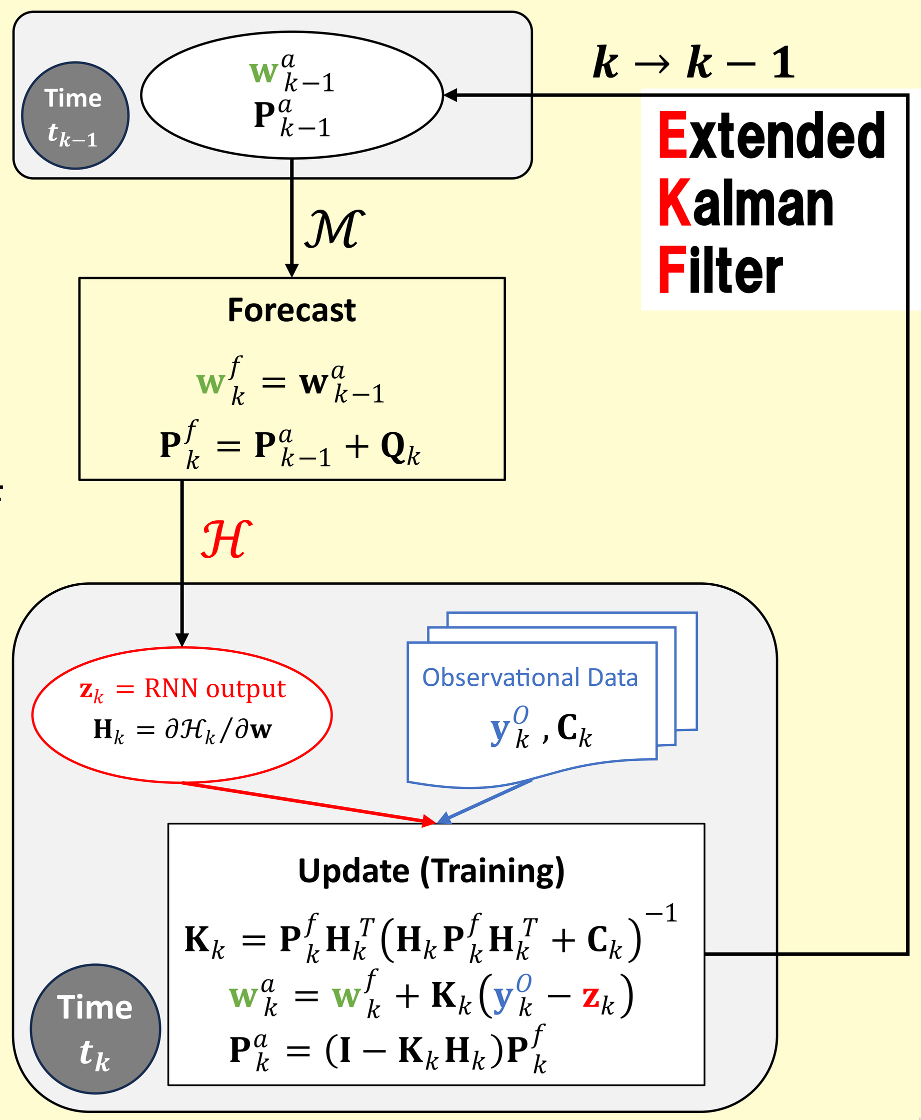
$$\mathbf{z}_k = \mathbf{W}^{(\mathrm{out})} \cdot \mathbf{h}_k + \mathbf{b}^{(\mathrm{out})}$$

⇧ Our machine learning method is based on an Elman network, which is a specific type of RNN that includes recurrent connections in the hidden layer $\mathbf{h}(t)$. In general, each node in a neural network mimics the behavior of a human perceptron by calculating a weighted sum s of its inputs v_i and applying an activation function ϕ.

The data learning procedure employed by our RNN ⇨ is based on the extended Kalman filter (EKF) algorithm for parameter estimation, as developed by Puskorius and Feldkamp (1994). The state vector, $\mathbf{w}$, consists of the components of trainable parameters in RNNs.

Unlike conventional backpropagation methods, the EKF dynamically updates the RNN weights by incorporating error covariance from training data, effectively mitigating overfitting while preserving computational efficiency. However, like any other neural network methods, our method is not free from initial-value problems, either. We had to make a grid search for the optimized initial state vector, $\mathbf{w}_0$, by changing the number of nodes, $D_\mathbf{h}$, in the hidden layer, $\mathbf{h}(t)$. The grid search finally yielded a combination of the best $D_\mathbf{h} = 34$ and the minimum variance of the estimated geomagnetic secular variation (SV) at Epoch 2020 for all 5-digit (2^5) initial vectors tried.

Extended Kalman Filter was used to form an interface between RNN and observation y^O with data covariance C.



Conclusions

- In this study, we utilized a new recurrent neural network to forecast the geomagnetic secular variation for the period from 2025 through 2030, employing an extended Kalman filter algorithm for training so as to give sufficient protection against overfitting problems.
- Our new machine learning method showed a better performance than our previous data assimilation method for predicting the short-term geomagnetic secular variation at Epoch 2020.
- We found that our neural network method trained with geomagnetic secular acceleration time series (viz., the second time derivatives of original Gauss coefficients) outperformed any other time derivatives (up to the fifth from the zeroth time derivatives), which implies that our machine learning method may have detected the secular acceleration variation due to Magneto-Coriolis waves in the Earth's outer core.

Results

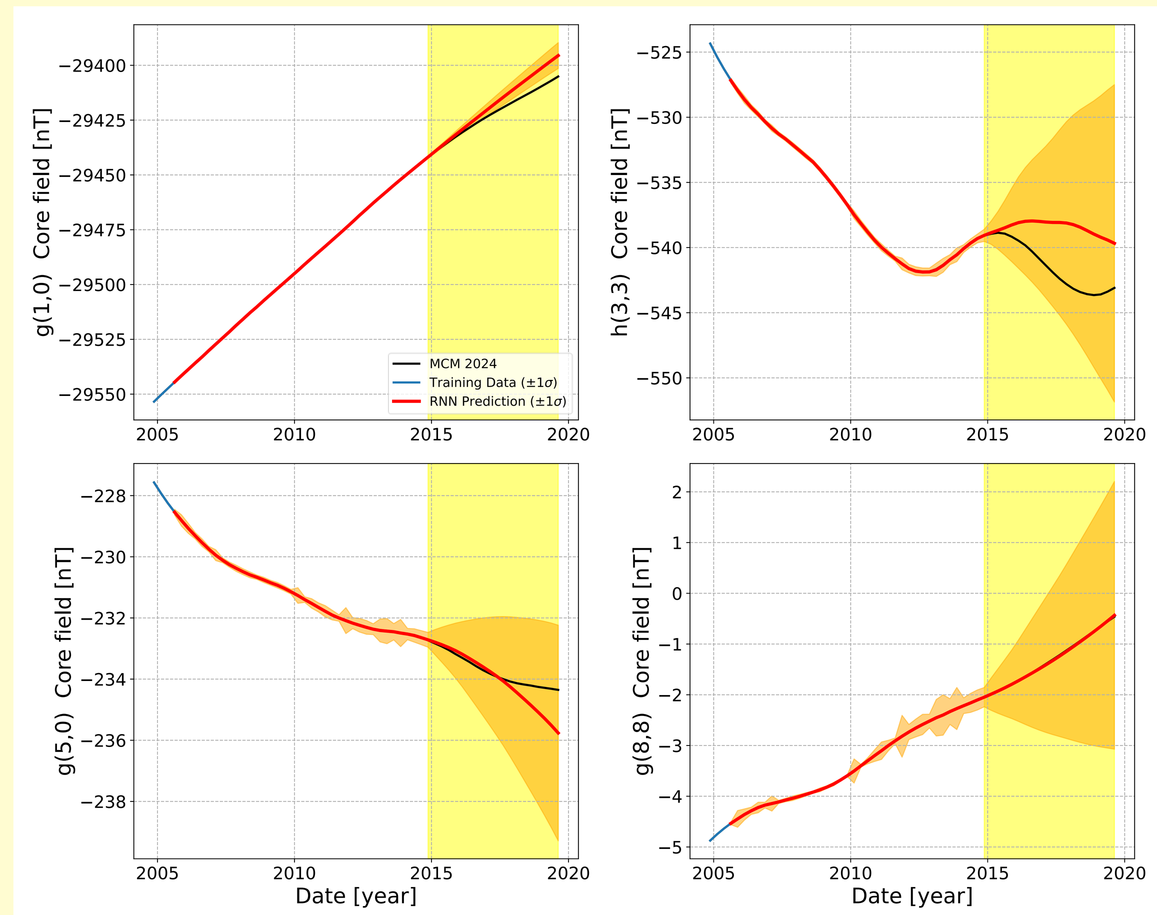
EKF-RN was found more effective for short-term forecast of SV than BP, 4DEnVar or IGRF.

Table 1. Summary of the hindcast experiments conducted in this study.

Scheme		Minimum $\sqrt{dP}$ 4.5 years after release [nT]	Median $\sqrt{dP}$ 4.5 years after release [nT]	Maximum $\sqrt{dP}$ 4.5 years after release [nT]
BPTT-RNN	50 iterations	72.5	77.9	84.3
	100 iterations	70.8	76.7	89.4
	500 iterations	85.4	86.4	91.0
	1000 iterations	85.8	87.3	92.3
EKF-RNN		70.3	78.0	84.1
4DEnVar (Minami et al., 2020)		100.9 Case (A4)	141.7	168.2 Case (A2)
IGRF-12		96.9	-	-

Table 1 clearly shows that EKF-RNN is superior to any other methods tried here (viz., back propagation, data assimilation and linear forecast by IGRF) in predicting short-term (i.e., ~ 5 yrs) SVs of our planet's main magnetic field. One strong point of our method is a better protection against overfitting problems in back propagation methods.

Hindcast results



The left panels show 4 time-series of Gauss coefficients for example. We used the MCM2024 field model by Ropp and Lesur (2023) as data (the black line), while training our EKF-RNN method from 2005 through 2015 to yield the red line. The beige envelope shows the range within ± 1 standard deviation. The yellow zone is our hindcast period, i.e., Year 2015 through 2020.

Because our new RNN method shows sufficient forecast ability for short-term SVs, we conducted SV forecast for Epoch 2025, which has already been submitted to IAGA IGRF-14 Task Force as a candidate SV model.

Acknowledgements

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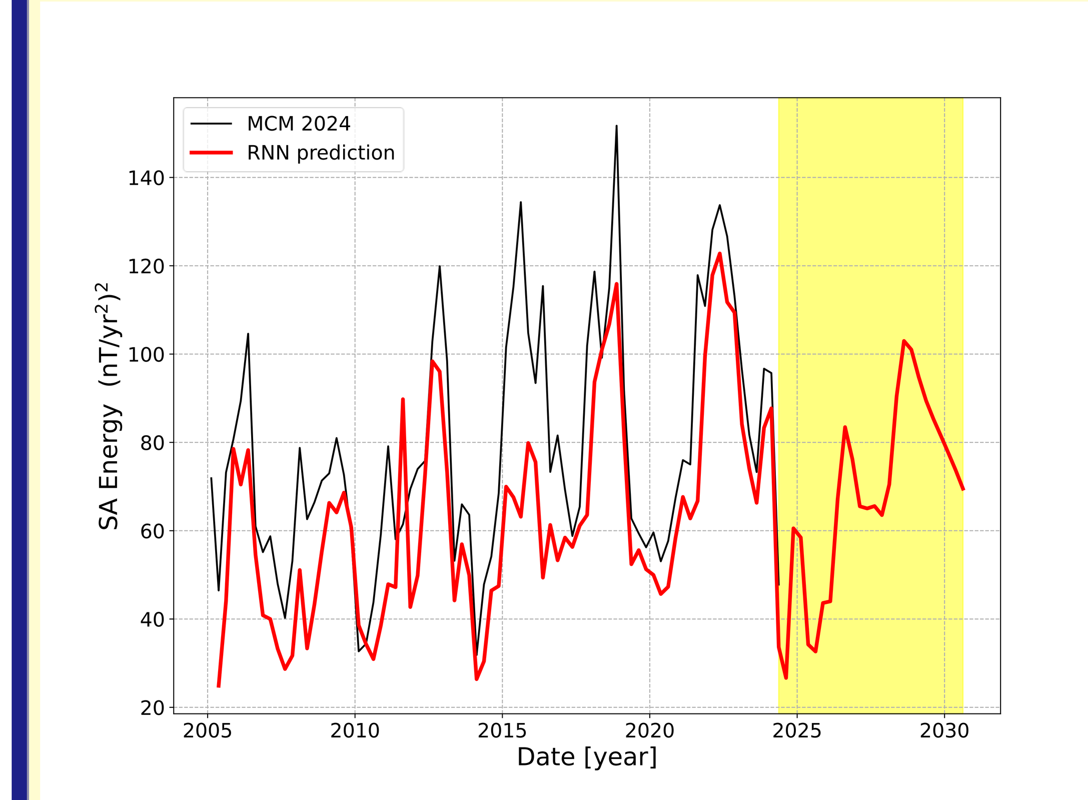
Discussion I:

Machine learning can detect periodic variations in the geomagnetic field.



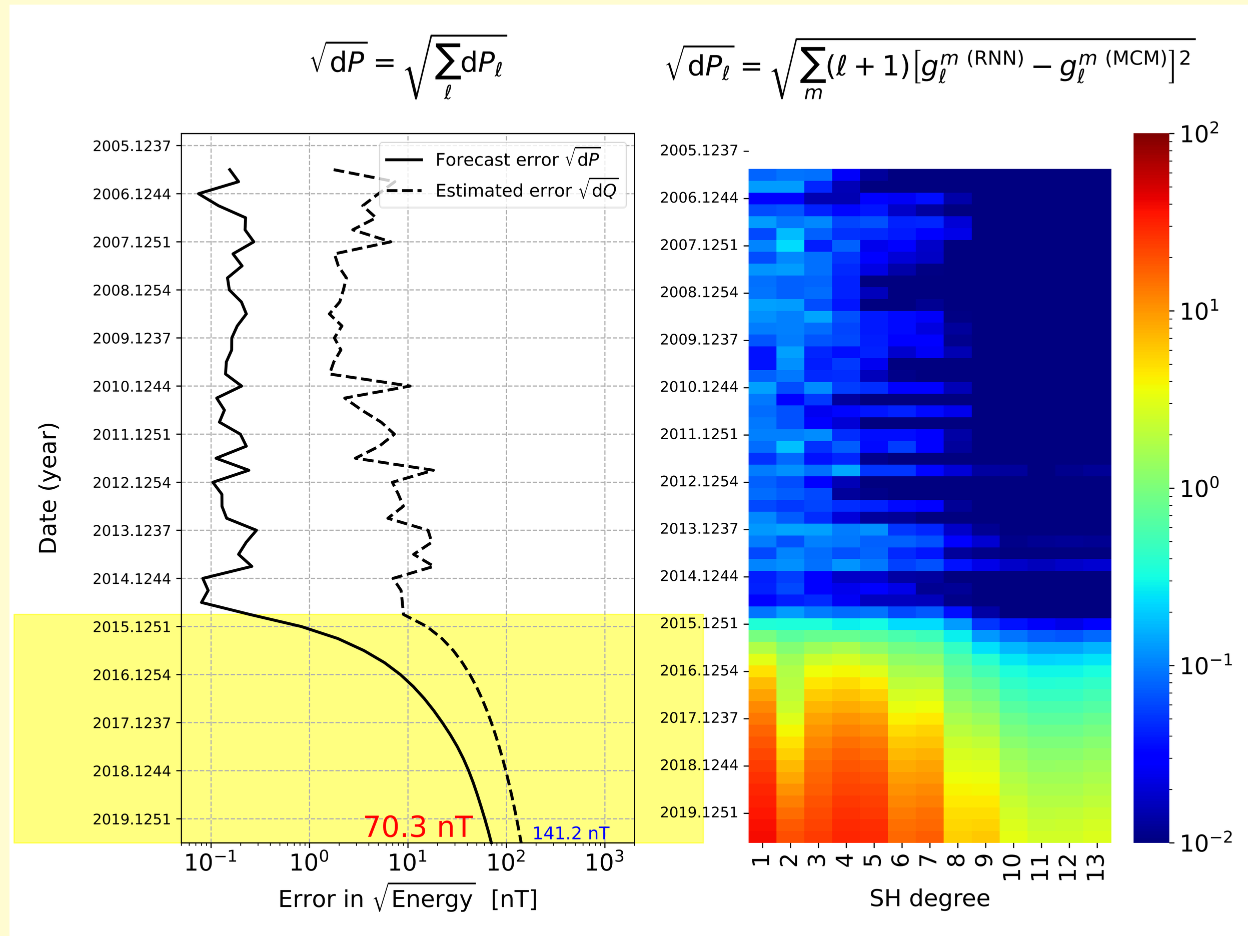
Prediction of the secular acceleration energy at the Earth's surface by our EKF-RNN method.

Our prediction for Gauss coefficients ends up with large forecast errors since we have worked with the second time derivatives rather than the Gauss coefficients themselves. However, it turned out that this is a necessary requirement by the data (i.e., the MCM2024 model) because the data contained a significant periodicity in its second time derivatives as shown in the left panel. Even though we have tried a variety of time derivative (zeroth through the fifith), the second time derivative gave us the best performance. The secular acceleration variation may be related to Magneto-Coriolis wave in the Earth's outer core (Gillet et al., 2022).

Discussion II:

Smaller forecast errors for higher-degree Gauss coefficients

It is quite natural that the forecast errors increase with time. However, they do possess degree-dependence because the higher degree Gauss coefficients vary more rapidly as shown in the right panels (except for Degree 2 in this case). This reflects the fact that core magnetic fields of short wavelength tend to have short time constants for their variations (Hulot and LeMouel, 1994).



Temporal evolution of the gap between observations ($g^{(\mathrm{data})}$) and RNN predictions ($g^{(\mathrm{RNN})}$) in terms of core field root mean energy ($\sqrt{dP}$). (b) Degree dependence of the gap ($\sqrt{dP_\ell}$).

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